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EDUCATIONAL PRODUCTION FUNCTIONS

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ABSTRACT

This research examines scholastic performance within the context of an individual's production function. A constant partial elasticity of substitution production function for academic achievement is presented and estimated with nonlinear maximum likelihood methods. We find that ability and time devoted to various aspects of the learning process are the most important determinants of students' accomplishments. Our results underscore the potential for students to compensate for relatively "poor" educational backgrounds by spending more time on study and class attendance.

INTRODUCTION

There exists in the social sciences a rich empirical and theoretical literature concerning the distribution of personal income. No matter what the approach, whether it be human capital (Mincer, 1974), statistical decomposition (Lydall, 1968), or eclectic socioeconomic analysis (Gintes, 1971), education surfaces as a prime quantifiable determinant of earnings differences within a population. This link between earning power and education underscores the importance of understanding the educational process.

Two basic views characterize the current state of economic analysis of education. First, education is treated as the output produced by a school. Second, the individual student is viewed as using his or her own time and effort, along with resources purchased from a school, to produce learning. Both approaches may be conceptualized by what economists call a production function. Economic theory places restrictions on the mathematical form of a production function, and we shall first examine briefly those restrictions. Next, we present estimates of educational production functions with data from a survey of undergraduate students at the University of North Carolina, Chapel Hill, corroborated by estimates from a national survey of college students.

Our research goals are to illuminate the value of the production function concept in helping to understand the educational process and to demonstrate that the production function concept provides some guidance in formulating public policy designed to influence the quality of education.

SCHOLASTIC PERFORMANCE: A PRODUCTION FUNCTION APPROACH

Consider an individual college student whose particular intelligence, amount of study time, and utilization of other resources leads to some level of academic success or scholastic performance. The mapping of this student's educational inputs ($\underline{X}$) into a degree of scholastic performance (y) is a production function

$$y = F(X), \qquad (1)$$

where $F(\cdot) \equiv$ the pruduction function.

In general, there are two mathematical properties of a production function worth noting. First, it is single valued, continuous, and well defined over the input set, yielding nonnegative outputs. Second, it has continuous first and second-order partial derivatives. (See Ferguson, 1969 chapter 4) for a useful background reference on the technical properties of production functions.) Figure 1 depicts a typical production function in two inputs. A first partial derivative of $F(\cdot)$ is known as a marginal product because it indicates the productive effect of increasing that input, all other input levels held constant. Positive marginal products will typically be observed, as negative marginal products indicate wasted resources.

A key property of equation (1) is that it indicates that a particular level of performance may be produced in a variety

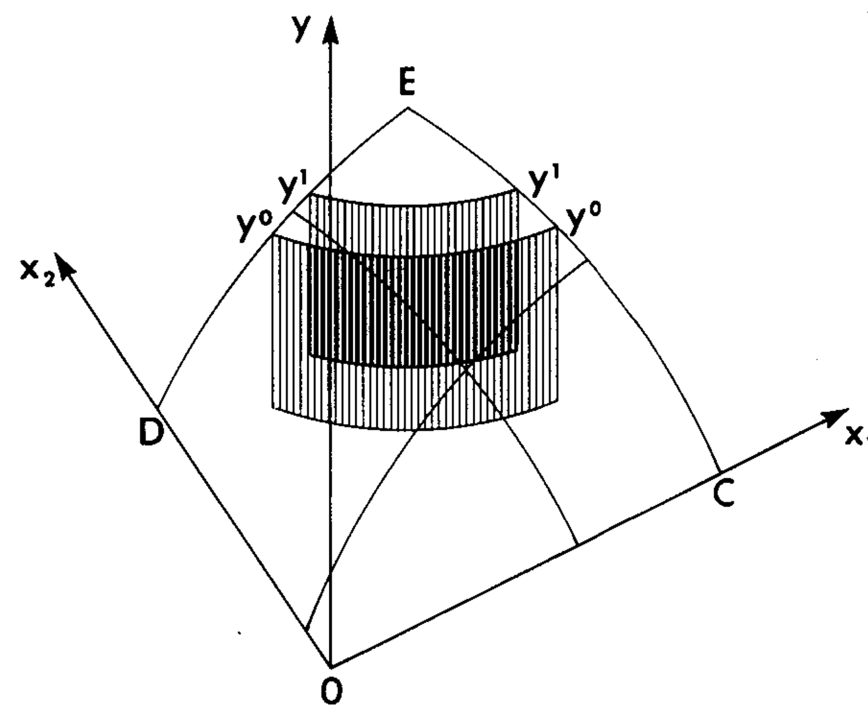

FIGURE 1

Production Surface ODEC Depicting Production Function for Output y with Two Inputs (X_1, X_2)

of ways. A grade of "B" may result from intensive home study coupled with sporadic class attendance or from perfect attendance paired with little study. What this means is that a student may trade-off class attendance for study time, or vice versa, in some proportion and still obtain a given grade. Such a trade-off is measured by a marginal rate of substitution (MRS). To see this characteristic of a production function, totally differentiate equation (1), set dy equal to zero, and solve for $(dX_j/dX_i)|_{\bar{y},\,\bar{X}_k;\,k\neq i,j}$ yielding

$$MRS \equiv \left(\frac{dX_j}{dX_i}\right)\Big|_{\bar{y},\bar{X}_k;\,k\neq i,j} = -\frac{\partial y/\partial X_i}{\partial y/\partial X_j}\,. \qquad (2)$$

Equation (2) gives us the increase in X_j necessary to hold y constant if X_i is decreased by a small amount. Figure 2 depicts two possible iso-output projections (isoquants), which are combinations of the inputs X_1 and X_2 that yield two particular levels of scholastic performance, y^0 and y^1. The isoquant labeled y^0y^0, for example, is obtained from Figure 1 by projecting into X_1,X_2 space all (positive) combinations of the two inputs that lie along a locus of constant height (y^0). The slope of an isoquant at a particular point, say A in Figure 2, is a graphical representation of the marginal rate of substitution of X_1 for X_2.

A standardized measure of the substitutability property is the elasticity of substitution (σ). This is defined as the percentage change in the ratio of two inputs resulting from a 1% change in the slope of the isoquant (marginal rate of substitution). In Figure 2, σ is the percent difference in the slope of ray OA and the slope of ray OB, divided by the percent difference in the slopes of isoquant (y^0y^0) at A and at B. The elasticity of substitution basically reflects the curvature of an isoquant. The easier it is to substitute one input for another, the greater the value of σ is, and the closer to a downward sloping straight line the isoquant is. "Perfect" substitution is said to exist when σ goes to infinity. At the other end of the spectrum is a production function whose isoquants are right angles (σ=0), indicating that inputs must be used in fixed ratios if waste is to be avoided. In general, production functions are classified according to whether σ is constant or varies along an isoquant. In our empirical research to follow, we make use of two different constant partial elasticity of substitution (CPES) production functions: the Cobb-Douglas and the generalized CPES (Cobb

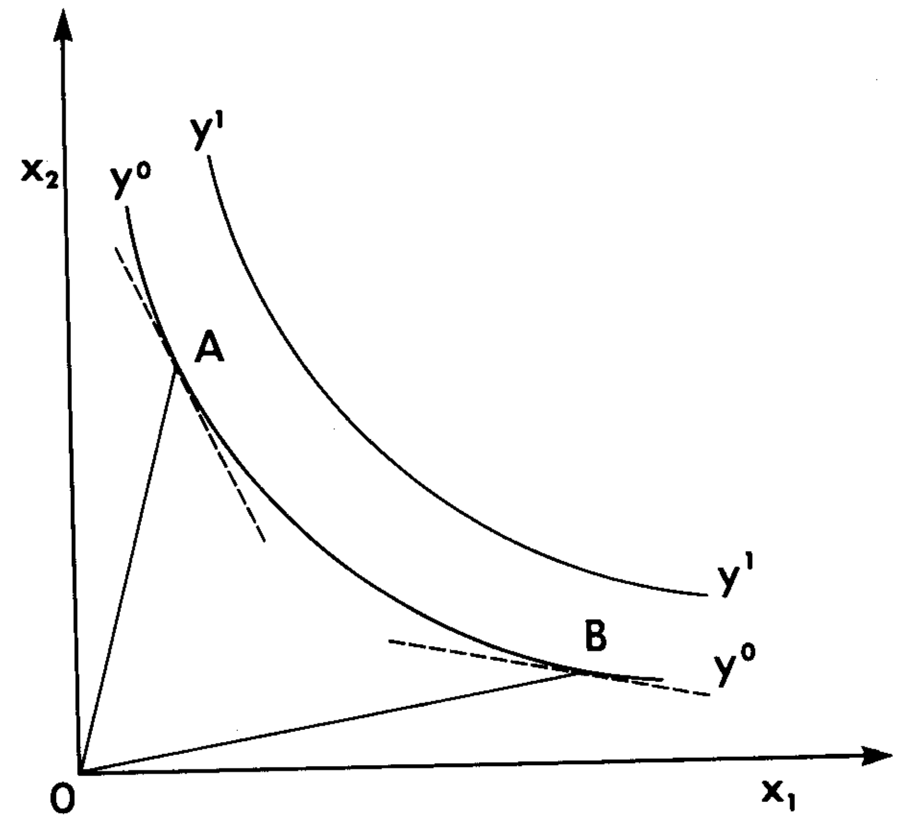

FIGURE 2

Iso-Output Projections of Input Combinations(X_1,X_2) Yielding Given Outputs y^0 and y^1

& Douglas, 1928). In the Cobb-Douglas function, σ is fixed at at 1.0, a priori, and in the CPES the data are permitted to indicate an elasticity of substitution that takes on the same (constant) value for all pairs of inputs. Whereas the functions we employ are not the most general for the purpose of examining the intricacies of the educational process, they do provide the basic insights we seek, while remaining relatively easy to estimate.

THE CPES PRODUCTION FUNCTION

Equation (3) is a CPES production function for academic success. This formulation was suggested by Uzawa (1962); it was selected because empirical experimentation with more complex functions (see Ferguson, 1969, p. 110) proved unfruitful.

$$y = \gamma\left[\sum_{i=1}^{n} \delta_i X_i^{-\rho}\right]^{-\mu/\rho}, \qquad (3)$$

where $y \equiv$ academic success
$X_i \equiv$ inputs of personal attributes, scholastic environment, and study time
$\gamma, \rho, \mu, \delta_i \equiv$ (positive) parameters
$i = 1,\ldots,n \equiv$ number of inputs.

This function has marginal products which are positive and diminishing:

$$\frac{\partial y}{\partial X_i} \equiv MP_i = \frac{y^{\frac{\mu+\rho}{\mu}}}{X_i^{\rho+1}} \mu\delta_i\gamma^{-\rho/\mu} \qquad i = 1,\ldots,n. \qquad (4)$$

From the equations summarized in expression (4), the marginal rates of substitution of X_i for X_j are

$$MRS_{ij} = \frac{\delta_i}{\delta_j}\left(\frac{X_j}{X_i}\right)^{\frac{1}{\sigma_{ij}}} \qquad i,j = 1,\ldots n, \; i \neq j. \qquad (5)$$

where $\sigma_{ij} \equiv \frac{1}{1+\rho}$.

From equation (5), it can be shown that σ_{ij} is the partial elasticity of substitution of X_i for X_j. To see this, differentiate equation (5) and solve for $\frac{d\ell n(X_j/X_i)}{d\ell n(MRS_{ij})}$, which is the definition of the elasticity of substitution. If one takes the limit of equation (3) as ρ goes to zero, the Cobb-Douglas production function $y = \gamma\prod_{i=1}^{n} X_i^{\delta_i}$ emerges. Production theory, as summarized in equations (4) and (5), serves as a guide in our subsequent empirical analysis of the marginal productivities of a student's resources and the potential to compensate for certain background deficiencies.

EMPIRICAL IMPLEMENTATION: THE CPES

Overview

This section is devoted to estimation of equation (3). We apply nonlinear maximum likelihood techniques to a unique set of data containing detailed information on student personal attributes, time allocation, and scholastic performance. We utilize test scores on a standardized midterm examination in a large lecture course to measure scholastic performance. (See Bowles, 1970, for a survey of this issue.)[1] Because the academic achievement of individual students within a given class setting is analyzed, broad measures of school quality are implicitly held constant. So, we examine mainly the roles of ability and time across students.

One of the peculiarities of an empirical study of a microeconomic educational production function is the paucity of data. No information exists on how much time students devote to study and class attendance. Because of such data limitations, we created a unique body of information by surveying students in the principles of economics course at the University of North Carolina at Chapel Hill. In addition, a national data set (Eckland, 1972, and Eckland & MacGillivray, 1972) that contains qualitative information is used to corroborate the empirical results from our survey.

At the University of North Carolina at Chapel Hill, the principles of economics courses are primarily taught with a lecture-seminar system. Students attend a large common

[1]Good grades have been shown by Wise (1975) to exert some positive influence on lifetime earnings independent of their tendency to encourage more education and thereby affect earnings indirectly. By contributing positively to a higher course grade, the production of a "good" examination score is consistent with greater future economic welfare for students.

lecture presented twice a week by a faculty member and a small one-hour seminar given by graduate teaching assistants. We surveyed 227 students taking a particular macroeconomic principles lecture plus one of several associated seminar sections during the spring semester, 1975. The period of the survey covers the part of the course (five weeks) between the beginning and first hourly examination. By limiting the survey to such a short period of time, we have minimized, it is to be hoped, measurement errors stemming from respondents' ability to recall class attendance and study time.

Data concerning sex, test score, study time, lectures attended, college board scores, and socioeconomic background were gathered; summary statistics are displayed in Table I. It should be noted that in conducting the survey we employed a double blind procedure which guaranteed that students involved could not be identified by name. One side-effect of this was that test reliability and validity measures could not be constructed.

Statistical Methodology

We utilize as the empirical counterpart of equation (3)

$$G_t = \gamma \left[\delta_1 L_t^{-\rho} + \delta_2 S_t^{-\rho} + \delta_3 C_t^{-\rho} \right]^{-\mu/\rho} + \varepsilon_t \qquad t = 1,\ldots,T \qquad (6)$$

where $t \equiv$ index of observation and
$\varepsilon_t \equiv$ random error term.

The dependent variable (G_t) is the proportion of correct responses on a 50-question objective test. This score reflects the learning that occurred during the first five weeks of the course and is one component of the student's course grade. L is the number of hours the student spent in class (lectures plus seminars) during the portion of the course prior to the examination. Study time (S) represents the total number of hours a student studied specifically for the first examination. S may best be interpreted as "cramming" time. The last of the three independent variables, C, is the individual's score on the quantitative portion of the SAT test, which is required of all UNC-CH students. To keep the empirical analysis as simple as possible at first, we are parsimonious in specifying the inputs in the grade production process. Later on, we control

TABLE I

Summary Statistics of UNC-CH Data

	The Sample Pooled			The Sample Stratified					
				Males			Females		
	Mean	Standard Deviation	Range	Mean	Standard Deviation	Range	Mean	Standard Deviation	Range
G	84.90	8.66	43	85.80	8.33	43	83.75	9.00	42
L	12.52	1.85	12	12.67	2.11	12	12.33	1.43	9
S	5.35	3.50	19	5.05	3.54	19	5.74	3.43	19
C	579.50	81.49	430	587.10	80.06	410	569.69	82.68	500

where G $\equiv$ numerical grade on midterm examination
L $\equiv$ lectures plus seminars attended
S $\equiv$ hours studied for the examination
C $\equiv$ score on quantitative section of Scholastic Aptitude Test

for sex and family background differences among students.

We assume that ε is independently and normally distributed with mean zero and constant variance σ^2. Equation (6) is <u>intrinsically</u> nonlinear, and we choose to estimate its parameters by nonlinear least squares.[2] This technique finds the values of the six parameters in equation (6) that minimize

$$S(\underline{\theta}) = \sum_{t=1}^{T} \{G_t - f(\underline{X}_t, \underline{\theta})\}^2, \tag{7}$$

where $T \equiv$ the number of observations

$\underline{\theta} \equiv [\gamma, \delta_1, \delta_2, \delta_3, \rho, \mu]'$ and

$\underline{X}_t \equiv [C_t, S_t, L_t]'$, where

$f(\cdot) \equiv$ production function

Given our assumption concerning the behavior of ε, the nonlinear least squares estimates of $\underline{\theta}$ are also the maximum likelihood.

We utilize the modified Gauss-Newton method for finding the values of $\underline{\theta}$ that minimize $S(\underline{\theta})$ (Draper & Smith, 1966, pp. 267-270).[3] The estimate of the variance of the errors ε_t is

$$S^2 = \frac{1}{T-6} S(\hat{\underline{\theta}}), \text{ where} \tag{8}$$

$\hat{\underline{\theta}}$ = values of $\underline{\theta}$ that minimize equation (6).

Gallant (1975, p. 74) shows that in large samples, $\hat{\underline{\theta}}$ has a six-dimensional multivariate normal distribution with mean $\underline{\theta}$ (true value)[4] and variance-covariance matrix $\sigma^2(F'F)^{-1}$, where F is the Tx6 matrix with elements

$$f_{tj} = \frac{\partial f(\underline{X}_t, \underline{\theta})}{\partial \theta j} \qquad t = 1, \ldots, T \quad j = 1, \ldots, 6. \tag{9}$$

In hypothesis testing, the matrix $(F'F)^{-1}$ must be approximated by the 6x6 matrix

$$\hat{D} = \left[F(\hat{\underline{\theta}})'F(\hat{\underline{\theta}})\right]^{-1}. \tag{10}$$

A $(1-\alpha)$ percent confidence interval for θ_j may be constructed as

$$\hat{\theta}_j \pm t_{\alpha/2}\sqrt{S^2\hat{d}_{jj}}, \tag{11}$$

where $t_{\alpha/2}$ is the $\alpha/2$ critical value of a t-distribution with T-6 degrees of freedom and $\hat{d}_{jj}$ is the j^{th} diagonal element of $\hat{D}$. From equation (11) the null hypothesis that $\theta_j = \theta_{j0}$ may be tested at the α percent level of significance by comparing

$$\left|\overset{\Delta}{t}_j\right| = \left|\hat{\theta}_j - \theta_{j0}\right| \Big/ \sqrt{S^2\hat{d}_{jj}}$$

with $t_{\alpha/2}$ and rejecting the null hypothesis when

$$\left|\overset{\Delta}{t}_j\right| > \left|t_{\alpha/2}\right|.$$

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[2]Equation (6) could also be estimated using a Taylor series approximation. See Kmenta (1967). However, the approximation for more than two inputs generates many collinear cross-product terms that make testing for parameter significance difficult. Direct nonlinear maximum likelihood estimation of equation (6) proved the most viable alternative.

[3]These estimates were also checked by applying the method of estimation known as Marquardt's compromise (Draper & Smith, 1966, p. 272). Little change (except for rounding errors) resulted.

[4]On this point, and for a more detailed presentation of the nonlinear least squares estimation technique, see Draper and Smith (1966, pp. 263-304). Also see Gallant (1975) for a concise presentation of hypothesis testing within the nonlinear regression framework.

Results

Equation (6) was estimated for the entire sample (T=227) using the maximum likelihood technique described above.[5] Regression (1) of Table II indicates an elasticity of substitution (σ) between factors of about 0.8.[6]

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[5]One a priori restriction was imposed when estimating equation (6):

$$\sum_{i=1}^{3} \delta_i = 1.$$

In this case, each δ_i is interpreted as a "share" of the ith input in the production of scholastic performance. On this point see Ferguson (1969, chapter 5).

[6]The parameters in Table II were calculated from a data matrix where lectures plus seminars (L) is scaled by a factor of 30 and hours studied (S) is scaled by a factor of 100. The differential magnification of these two independent variables facilitated calculation of the standard errors of the regression parameters. To see this, remember that the standard errors are the square roots of the diagonal elements of the matrix $\hat{\sigma}^2 \left[F(\hat{\underline{\theta}})' F(\hat{\underline{\theta}})\right]^{-1}$ and that

$$f_{t\delta_i} \equiv \frac{\partial f(\underline{X_t},\underline{\theta})}{\partial \delta_i} \quad (t = 1,\ldots,T)$$ is one of the column vectors in the Tx6 matrix F. (See equations (8) and (9) of the text for definitions of $\underline{\theta}$, f(·), and F.) Further, notice that for the production function given by equation (6),

$$\partial f(\cdot)/\partial \delta_i = -\frac{\mu\gamma}{\rho}\left[\cdot\right]^{-\left(\frac{\mu+\rho}{\rho}\right)} X_i^{-\rho} \quad (i = 1,2,3).$$

Since ρ is small, the raw values of L and S generate little covariation between $f_{t\delta_1}$ and $f_{t\delta_2}$, and thus lead to a poorly conditioned F'F matrix. When L and S are rescaled, this problem is eliminated.

TABLE II

Nonlinear Least Squares Estimates
CPES Production Functions

	Parameter Estimates[a]						Computed Marginal Products[b]		
	γ	δ_1	δ_2	δ_3	ρ	$\frac{\mu}{\rho}$	$\overline{MP}_L$	$\overline{MP}_S$	$\overline{MP}_C$
(1) The Sample Pooled T = 227 SEE = 8.48	6.05 (2.61)	0.10 (1.26)	0.05 (2.62)	0.84 (10.90)	0.24 (2.25)	1.74 (2.09)	0.72	1.04	0.05
(2) The Sample Stratified Males T = 128 SEE = 8.51	8.14 (1.70)	0.17 (1.08)	0.04 (1.09)	0.79 (4.75)	0.16 (0.14)	2.32 (0.14)	0.78	0.54	0.04
(3) Females T = 99 SEE = 7.38	4.97 (1.57)	0.00 (0.00)	0.08 (1.09)	0.94 (6.19)	0.19 (0.09)	2.28 (0.09)	0.00	1.24	0.06

Production Function: $G = \gamma\left[\delta_1 L^{-\rho} + \delta_2 S^{-\rho} + \delta_3 C^{-\rho}\right]^{-\frac{\mu}{\rho}}$

[a]t-values in parentheses

[b]$\overline{MP}_L$, $\overline{MP}_S$, $\overline{MP}_C$ ≡ marginal products evaluated at mean values (unscaled)

For the complete sample, the marginal product of attending one extra lecture or seminar is an increase of about .7 points on the midterm examination. This is 25% less than the marginal product of an hour of study time. Finally, an extra 100 points on the quantitative section of the SAT leads to an exam score higher by approximately 5 points.[7] From this information we calculate marginal rates of substitution and present them in Table III.

Our results indicate that less able students can indeed compensate for differences in ability with extra study and class attendance. For example, a total of about 7 additional hours of study (about 1½ hours per week for the five week period used here) are necessary to offset a 100 point SAT score disadvantage. If a student were taking four courses with similar learning structures, this result implies that an extra 6 (12) hours per week of study compensate for a 100 (200) point SAT deficiency. Such variations in SAT scores are typical of state universities. The associated study time requirements, while substantial, are not unreasonable. By examining the rest of Table III, one can see the other possible tradeoffs in the production of scholastic performance.

One interesting question that our data permit us to address is whether the possibility of compensating for background deficiency with extra class attendance or study time differs between male and female students. We thus stratify the sample and estimate regression equation (6) separately for males and females. Stratification yields samples of 128 male and 99 female students. Little difference exists between the sexes with respect to μ and ρ, but large differences exist for γ and δ. Females have a smaller γ, implying that on average they score less than males, *ceteris paribus*. The sex differences in parameters in Table III imply that females have a higher marginal product of study and a lower marginal product of class attendance. One possible interpretation of these parameter differences by sex is that the test instrument meters to

[7]The most widely used measures of ability of college students are the verbal and quantitative college board examination, though these scores may capture additional factors. It best served our purpose here to employ the college board quantitative score by itself as an ability proxy, as it is a better independent predictor of exam score than the verbal score alone or the sum of the scores.

TABLE III

Marginal Rates of Substitution
Derived from the CPES Production Function
Estimates of Table II[a]

		L	S	C
Total Sample N = 227	L	-	0.70	13.8
	S	1.43	-	19.8
	C	0.07	0.05	-
Male Sample N = 128	L	-	1.46	18.1
	S	0.68	-	12.6
	C	0.05	0.08	-
Female Sample N = 99	L	-	b	b
	S	b	-	20.2
	C	b	0.05	-

[a]In the matrix of marginal rates of technical substitution [S], the element S_{ij} represents the increase in the j^{th} input required to hold output constant when the i^{th} input is decreased by one unit.

[b]$MP_L = 0$

some extent memorization ("study") of facts, while class is devoted to concepts. If females are better at memorization than at concept formation (a popular prejudice), and the test weights facts and concepts about equally, then it is reasonable to find that females get relatively more product from study time and males from class time. (We are indebted to an anonymous referee for pointing this out.) Given their higher relative productivity in study, then, efficient allocation of resources within the educational process requires relatively more study time for females. The mean values of S and L in Table I are consistent with this.

CORROBORATION OF EMPIRICAL RESULTS: THE COBB-DOUGLAS FUNCTION

Because of the admitted narrowness of the UNC-CH data, we now examine in some detail the robustness of the empirical results in Table II. We utilize a national data set (Eckland,1972), which is a detailed follow-up study (performed in 1970) of a group of high school seniors originally surveyed in 1955 by the Educational Testing Service (ETS). The follow-up is a stratified sample of 42 of 516 originally surveyed schools and was selected to provide a proportionate representation of schools across the United States. The Eckland data contain detailed student information on high school courses and performance (including the number of math and science courses taken), an objective measure of ability, and self-ratings of intelligence, diligence, creativity, and intellectual confidence. Student background variables include parental education and income as well as number of siblings and a host of attitudinal questions. Data are also available on freshmen through senior performance (grade-point averages in college and in major field of study).

Despite a broad scope, the Eckland data contain some deficiencies. Certain key variables of particular interest to our research take on only a limited number of values and thus complicate interpretations. For example, ability (APT) is measured as a score on a 20-question test, and quantitative ability is not separated from verbal ability. Only proxies exist for other ability components. Diligence (DILEG) is measured retrospectively on a 4-point scale, and no quantitative information exists for actual time spent studying in and out of class. Given this lack of precise measurement, only qualitative implications with respect to the production of scholastic performance can be drawn. So, rather than employ the computationally cumbersome (expensive) CPES production function, we estimate Cobb-Douglas production

functions via ordinary least squares[8] to check the qualitative aspects of our results in section 4.

Regressions with freshman grade-point average and overall college grade point average as the dependent variables are presented in the first half of Table IV. These regressions are consistent with our earlier findings that (a) ability and (b) diligence or time intensity of study are key determinants of scholastic performance.[9]

In particular, the coefficients of personal attribute variables are not statistically significant, while those of ability and diligence measures are. Finally, to guard against the possibility that the results in Tables II and IV are similar because of the change in functional form of the regression equation, we estimated the Cobb-Douglas specification with the UNC-CH data. This regression is also presented in Table IV. Ability and diligence factors again dominate background and personal attribute measures as explanatory variables. The results in Table IV illustrate the (qualitative) robustness of the CPES parameter estimates of section 4.

A QUALIFICATION

One qualification is necessary. Learning in one course represents an extremely narrow aspect of the entire learning process within the university. In reality, learning should be viewed as a larger problem in which resources must allocate to the production of grades in a number of competing courses. Viewing learning and time

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[8]A logarithmic transformation of the Cobb-Douglas function ($y=\gamma\prod_i X_i^{\delta_i}$) yields an estimating equation $\ln y=\ln\gamma + \Sigma\delta_i \ln X_i$. We assume an error term that is independently and normally distributed with mean zero and constant variance. SEX, ATTRAC, AND S*AT are added as control variables.

[9]The Eckland data regressions were also run without an adjustment for high-school grade-point average (GPAHS). Little difference in parameter estimates resulted. The regressions with GPAHS are reported because such a specification is thought to represent a more appropriate scholastic performance production function.

TABLE IV

Scholastic Performance Production Functions (Cobb-Douglas OLS Specification)

	ECKLAND DATA				UNC-CH DATA			
	Freshman GPA		Overall GPA		Examination Score			
Independent Variables	coef	t-value	coef	t-value	coef	t-value	Mean Values	Marginal Products
Ability:								
CBQ					0.230	4.44	583.53	.033
CBV					0.164	3.32	539.47	.026
APT	0.023	3.70	0.011	2.11				
CREAT	-.066	-2.13	0.016	0.59				
INTEL	0.028	0.55	0.122	2.77				
INTCON	0.085	1.99	0.067	1.82				
GPAHS	0.341	8.04	0.257	7.09				
Diligence:								
SEM					0.067	2.21	3.69	1.54
STUDY					0.025	2.13	5.11	.416
LCT					0.034	0.58	8.38	.345
WKST					-.005	-0.44	2.25	- .189
DILEG	0.386	11.53	0.207	7.25				
Background:								
FINC					0.009	0.88	27.52	.028
FED					0.018	0.67	14.61	.105
MED					-.002	-0.06	14.02	-.012
Personal Attributes:								
SEX	-.202	-1.48	-.125	-1.07	-.024	-1.67	0.40	-5.096
ATTRAC	-.068	-1.30	-.028	-0.63				
S*AT	0.070	1.02	-.007	-0.12				
No. Observations	655		655		227			
$\bar{R}^2$	0.35		0.30		0.31			

TABLE IV (continued)

CBQ ≡ score on college board (SAT) quantitative examination range: 200-800

CBV ≡ score on college board (SAT) verbal examination range: 200-800

APT ≡ score on ETS aptitude test range: 0-20

CREAT ≡ respondent's rating of own creativity range: 4 (very creative) -1 (not at all creative)

INTEL ≡ respondent's rating of own intelligence range: 4 (very intelligent) -1 (not at all intelligent)

INTCON ≡ respondent's rating of own intellectual confidence range: 4 (very) -1 (not at all)

GRAHS ≡ high school grade-point average range: 1 (F) -5 (A)

SEM ≡ number of seminar sessions attended

STUDY ≡ number of hours of study for midterm examination

LCT ≡ number of lectures attended

WKST ≡ average study hours per week

DILEG ≡ diligence measured by whether respondent completed class assignments and recommended reading range: 4 (almost always) -1 (rarely or never)

FINC ≡ family income ($000's)

FED ≡ father's level of education

MED ≡ mother's level of education

SEX ≡ dummy variable 0 (male) -1 (female)

ATTRAC ≡ respondent's rating of attractiveness to opposite sex 3 (very) -0 (not at all)

S*AT ≡ interaction of SEX and ATTRAC

Dependent Variables
Eckland Data: GPA measured on a four point scale
UNC-CH Data: midterm examination score on 100 point scale

Functional Form:

$$\ln Y = \alpha_0 + \sum_{i=1}^{n} \alpha i \ln X_i + \sum_{j=1}^{m} \beta_j Z_j ,$$

where $\ln Y$ is the $\log_e$ of the dependent variable, $\ln X_i$ is the $\log_e$ of the ith input into the production of scholastic performance, and Z_j is the jth conditioning factor.

allocation within the narrow context of one course may lead to biases in our coefficient estimates analogous to simultaneous equations bias. Whereas in our model each input is taken as independent, a more rigorous specification would allow for interrelation between inputs and desired grades across a student's courses. Lack of data in the UNC-CH survey prohibits the construction of such simultaneous models.

SOME POLICY IMPLICATIONS

In this research we view scholastic performance within the context of an individual's production function. We find that innate ability and acquired background are the most important determinants of students' accomplishments. Moreover, time devoted to the learning process proved significant. In particular, class and seminar attendance, as well as study time, yield positive (but differing) marginal products. Estimates of marginal products are potentially useful in allocational decisions within the educational process. For example, suppose that an administrator has X dollars to spend on an activity that improves students' academic performances. Suppose also that by spending this amount on activity A, students each attend one additional class, whereas by spending this amount on activity B, students each study one additional hour. Our hypothetical administrator will make better use of his or her resources by choosing the activity with the higher marginal product.

Our findings also identify some trade-offs within the educational process. For example, we find that approximately 1½ extra hours of study per week compensate for a 100-point college board score deficiency. This result underscores that it is possible for a diligent student with a poor educational background to be academically successful, a fact sometimes neglected by school administrators when formulating admission standards. Finally, our production function estimates illustrate the significance of students' own endeavors in achieving academic success, a factor typically neglected when analyzing the social value of education. Part of the benefits often attributed to public expenditure are direct outputs of student effort.

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